Suppression of Zeeman splitting and polarization steps in localized exciton-polariton condensates

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We show that the condensation of exciton polaritons in semiconductor microcavities in an applied magnetic field manifests itself in the quenching of the Zeeman splitting of an elliptically polarized condensate. The circular polarization degree of a localized condensate with a finite number of particles increases as a function of the magnetic field with a steplike behavior. The width of each polarization step is fixed by the polariton-polariton interaction constants and the number of steps is fixed by the number of polaritons in the condensate. The magnetic susceptibility of the condensate depends qualitatively on the parity of the number of polaritons.

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I. INTRODUCTION

The Bose-Einstein condensation (BEC) of exciton-polaritons in semiconductor microcavities has been recently reported.[1,2] This discovery opens the way to the realization of coherent matter states at unusually high temperatures and multiple applications, including polariton lasers.[3] An essential peculiarity of polaritons in planar semiconductor microcavities containing quantum wells consists of the presence of a polarization degree of freedom. A linear polarization is expected to build up spontaneously as a result of polariton BEC[4] or in the superfluid phase.[5] This effect can be understood by assigning the pseudospin $+1(-1)$ to each polariton, which corresponds to the right (left) circular polarization. Polaritons with the same pseudospin repel each other, while there is weak attraction between the polaritons with opposite pseudospins. As a result, the energy of a polariton system is minimized when equal numbers of left and right circularly polarized polaritons are present. This implies a linear polarization for a polariton system in a condensed or superfluid state.

It was recently shown that an applied magnetic field has a pronounced effect on the polarization state of superfluid polaritons.[6] In this case there is a competition between the magnetic field that tries to make the polariton system circularly polarized and the polariton-polariton interaction that favors a linear polarization. In weak magnetic fields this competition is resolved in the formation of an elliptically polarized superfluid and the complete suppression of the Zeeman splitting. Only when the magnetic field exceeds a critical value does the polarization become circular and the Zeeman splitting appears.

While the superfluidity of exciton polaritons remains still a theoretical prediction, the formation of localized polariton condensates has been observed, both in random[1,7] and artificially prepared[2] traps. Localized polariton condensates were also created by resonant optical excitation in a planar microcavity having a submicron scale potential disorder[8] and the possibility of creating polariton condensates in micropillar microcavities is an area of active research.[9]

In this paper we analyze the magnetic field effects on such a finite-sized polariton system, or a *polariton dot*, that can contain a small number of polaritons, N. We assume that polaritons behave as ideal bosons and the temperature is low enough such that they occupy the same orbital quantum state, i.e., they form a condensate within the dot. We consider an effect of the external magnetic field parallel to the structure axis on the energy spectrum and spin polarization of exciton polaritons. We ignore for simplicity the field effect on orbital motion of elections and holes which results in Landau quantization. This effect has been considered in detail by Lozovik and co-authors.[10] It is not sensitive to spin and polarization of polaritons which are in the scope of the present work. On the other hand, we fully take into account the Zeeman effect. We assume that each exciton polariton can be characterized by a g factor given by the exciton g factor renormalized accounting for the Hopfield coefficients.[11] Experimentally, the magnetic field effect on the linear optical response of exciton polaritons has been studied in detail,[12,13] while no experimental results on polariton BEC under magnetic field have been reported so far, to the best of our knowledge.

II. THE POLARITON DOT IN A MAGNETIC FIELD

The Hamiltonian of our model system reads

$$H = \frac{1}{2}\sum_{\sigma=\pm}\left[\alpha_1 N_\sigma(N_\sigma - 1) + \alpha_2 N_\sigma N_{-\sigma} - 2\sigma\Omega N_\sigma\right], \quad (1)$$

where the energy of noninteracting polaritons at zero magnetic field is taken to be equal to zero, $\sigma=\pm$ is the pseudospin index defined in such a way that N_+ is the number of polaritons with spins parallel to a magnetic field, and $N_- = N - N_+$ is the number of polaritons with spins antiparallel to the field. The bare Zeeman splitting is given by $2\Omega = g\mu_B B$ where B is the applied magnetic field, g is the polariton g factor, and μ_B is the Bohr magneton. α_1 and α_2 are the polariton-polariton interaction constants in triplet (parallel spins) and singlet (antiparallel spins) configurations,

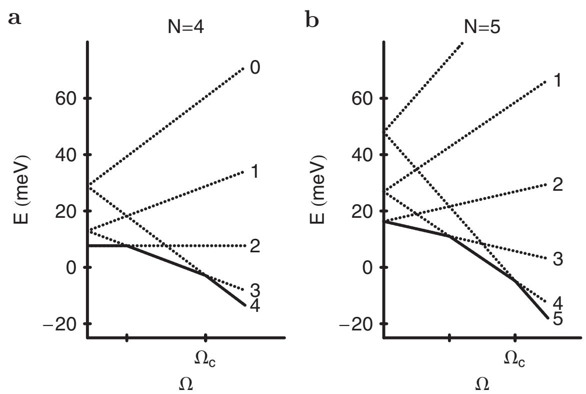

FIG. 1. Dotted lines show the energy levels for an even (a) and odd (b) number of particles. The energy levels are labeled by the number N_+. Solid lines show the ground state. We used the parameters $\alpha_1=4.8$ meV (assuming the dot has a diameter of about 100 nm, i.e., the dot is 20 times smaller than the one in Ref. 7) and $\alpha_2=-0.1\alpha_1$ (Ref. 15).

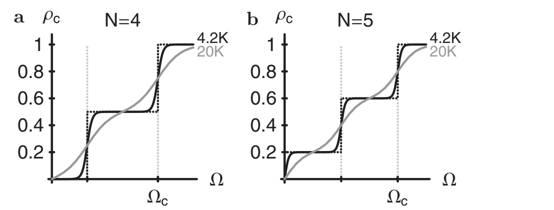

FIG. 2. The dotted black line shows the circular polarization degree ρ_c at $T=0$ for an even (a) and odd (b) number of polaritons. The solid lines correspond to the finite temperatures 4.2 K (black) and 20 K (gray).

respectively.[14] The Hamiltonian (1) is already diagonal and the energy levels $E=E(N,N_+)$ can be labeled by two quantum numbers, N and N_+. These levels are shown in Fig. 1.

It is seen from Fig. 1 that there are abrupt changes of the ground state of the system as the field is increased. Each abrupt change corresponds to the reorientation of one polariton pseudospin, which increases the number N_+ by 1. For an even total number of polaritons, the ground state does not change at very low fields and $N_+=N/2$. Then at $\Omega=(\alpha_1-\alpha_2)/2$ the state with $N_+=(N/2)+1$ obtains a low enough energy to become the ground state. Next, at $\Omega=3(\alpha_1-\alpha_2)/2$, the state with $N_+=(N/2)+2$ becomes the ground state, and so on, until all polaritons align their spins $(N_+=N)$ at the critical field B_c given by

$$\Omega_c=\frac{1}{2}g\mu_B B_c=\frac{1}{2}(N-1)(\alpha_1-\alpha_2). \tag{2}$$

A similar behavior appears for an odd number of polaritons, however, the values of the field at which changes occur are shifted by $(\alpha_1-\alpha_2)/2$. The critical field, when all polariton spins are aligned, is still given by Eq. (2).

The ground state energy decreases linearly with the field, but each change in N_+ causes an abrupt increase in the magnetic susceptibility (Fig. 1). The overall decrease for $\Omega<\Omega_c$ is close to parabolic for large values of N. For $\Omega>\Omega_c$ the ground state energy is given by

$$E_0(\Omega)=-\Omega N+\frac{\alpha_1}{2}N(N-1). \tag{3}$$

These changes of the ground state lead to steps in the circular polarization degree ρ_c, which is defined as

$$\rho_c=\frac{1}{N}\langle N_+-N_-\rangle=\frac{2\langle N_+\rangle}{N}-1. \tag{4}$$

Here the brackets indicate averaging over different states of the polariton system. First we consider the case of a fixed number of polaritons in the dot and assume that polaritons are in quasithermal equilibrium with an effective temperature T. In this case the averaging is performed with a usual thermal distribution, i.e., the probability to observe the state with energy $E(N,N_+)$ is

$$p(N_+)=Z^{-1}e^{-E(N,N_+)/(k_BT)}. \tag{5}$$

Here Z is the partition function found from the normalization $\Sigma_{N_+}p(N_+)=1$. The polarization steps are shown in Fig. 2. Note that the low-field susceptibility is sensitive to the total number of particles. At low temperatures it is close to zero for an even number of polaritons, and it is described by the spin-1/2 Brillouin function for an odd number of polaritons.

There is an interesting analogy between the polarization steps we discuss in this paper and the magnetization steps observed in the magnetization curve of II–IV diluted magnetic semiconductors in high magnetic fields (see, e.g., Ref. 16 and references therein). The magnetization steps appear due to the change of the ground state of antiferromagnetic pairs of Mn^{2+} ions. At small fields the ground state of a $Mn^{2+}-Mn^{2+}$ pair has a total spin $S=0$ and its projection $S_z=0$. The magnetization steps correspond to the subsequent changes of the ground state to $S=1$, $S_z=-1$, then to $S=2$, $S_z=-2$, etc., with increasing magnetic field, until the fully polarized state with $S=5$, $S_z=-5$ becomes the one with the lowest energy. In general, the states of a dot with an even number N of polaritons can be mapped to the states of an antiferromagnetic pair of two $N/4$ spins. It should be noted, however, that the Hamiltonians of these systems are different (such that the mapping does not have a one-to-one correspondence). In particular, they have different numbers of eigenstates. There are $N+1$ states in the case of a polariton dot, and $(N+2)(N+2)/4$ states in the case of two $N/4$ spins.

As in the case of magnetization of antiferromagnetic pairs,[17] there are several mechanisms of broadening of polarization steps apart from the temperature broadening. Below we consider two most important ones. Namely, these are the finite polariton lifetime τ_p and fluctuations of the number of polaritons within the dots. Note that the finite lifetime is not negligible even if $\hbar/\tau_p$ is much less than the polariton-polariton interaction constants. This is because each step corresponds to the crossing of two levels with different circular polarizations, and the broadening of polariton levels is certainly important in the vicinity of the crossing points. The effect of fluctuations of the polariton number is more interesting. The positions of the steps depend only on the parity of the number of polaritons N, so that both even and odd

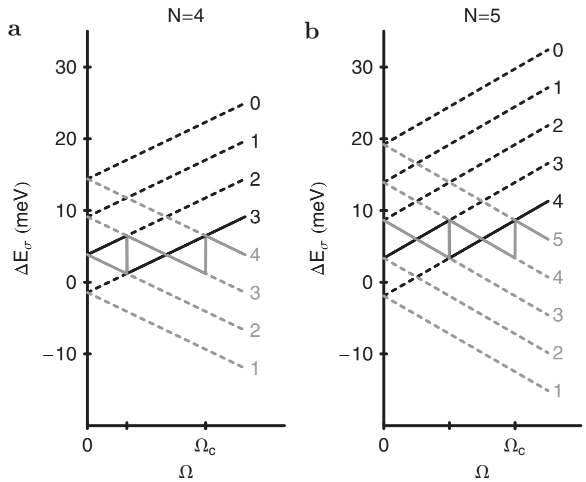

FIG. 3. Black and gray lines show the energy changes, ΔE_- and ΔE_+, for the emission of a photon with σ_- and σ_+ polarization, respectively. Solid lines represent the energy changes from the initial ground state, while dashed lines show the possible energy changes from excited states. (a) An even number of polaritons in the initial state; (b) an odd number. Each transition is labeled by the initial value of N_+.

steps are expected to be observed. On the other hand, the amplitude of each step is inversely proportional to N or $N+1$ for an even or odd number of polaritons, respectively. Therefore fluctuations in N also produce an additional broadening of the steps as a result of statistical averaging.

III. PHOTOLUMINESCENCE IN A MAGNETIC FIELD

The presence of polarization steps discussed in the previous section is reflected in the photoluminescence (PL) spectra of the polariton dot. In a typical PL experiment the polarization dependence of the energy of photons emitted from the dot is observed. The energy of the photon depends on the initial state of the polariton system and can be written as

$$\hbar\omega = \hbar\omega_0 + \Delta E_\sigma, \tag{6}$$

where ω_0 is the bare single polariton frequency,

$$\Delta E_+(N,N_+) = E(N,N_+) - E(N-1,N_+-1), \tag{7a}$$

$$\Delta E_-(N,N_+) = E(N,N_+) - E(N-1,N_+). \tag{7b}$$

The energy differences ΔE_σ are shown in Fig. 3. It is seen that there is a pronounced suppression of the Zeeman splitting of the emission from the ground state for magnetic fields below the critical field Ω_c. A noticeable Zeeman splitting remains, however, for the excited states. Therefore it is important to analyze the behavior of PL lines at a finite effective temperature.

To calculate the emission spectrum we introduce a linewidth broadening defined by $\Gamma = \hbar/\tau_p$. Then, the probability of the system to emit a σ-polarized photon with energy $\hbar\omega = \hbar\omega_0 + \varepsilon$ is

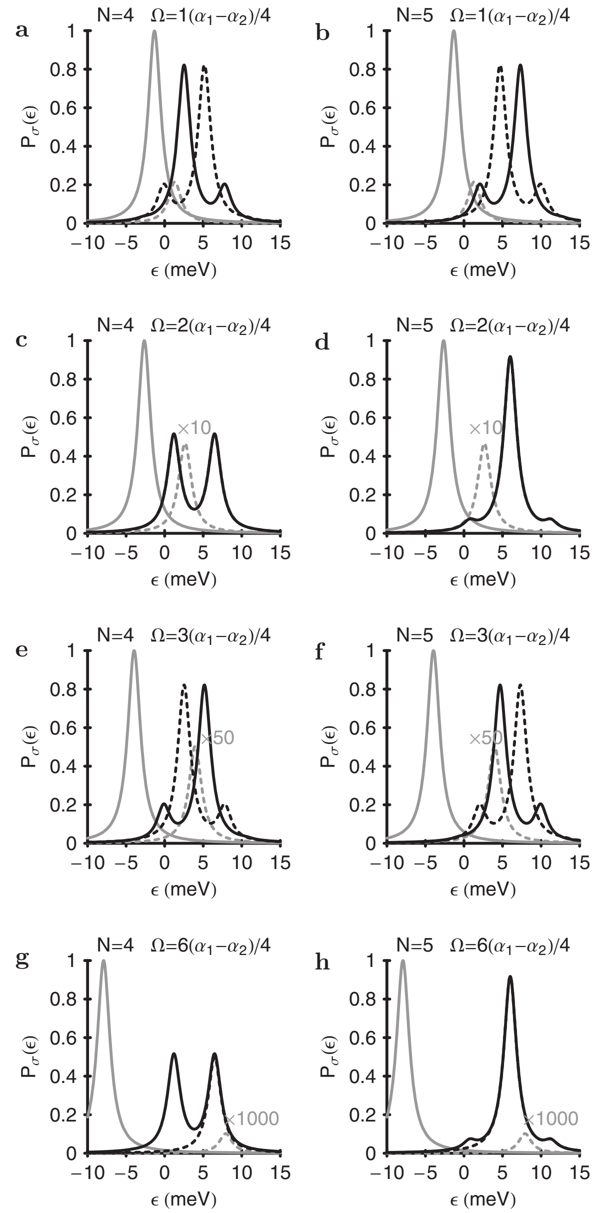

FIG. 4. PL spectra of σ_+ emission (solid curves) and σ_- emission (dashed curves) for an even (left) and odd (right) fixed number of polaritons for different magnetic fields. To illustrate the screening of the Zeeman splitting, gray curves show the spectra in the absence of polariton-polariton interactions. We used the parameters: $T = 20$ K, $\Gamma = 1$ meV.

$$P_\sigma(\varepsilon) = \frac{1}{Z}\sum_{N_+} \frac{p(N_+)\Gamma^2}{[\Delta E_\sigma(N,N_+) - \varepsilon]^2 + \Gamma^2}. \tag{8}$$

The PL spectra obtained from Eq. (8) are plotted in Fig. 4.

When Ω is an integer multiple of $(\alpha_1-\alpha_2)/2$ and $\Omega < \Omega_c$, peaks in the σ_+ and σ_- emission overlap, i.e., for special values of the field there is complete suppression of the Zeeman splitting. From Fig. 3 we note that at these values of Ω there is a crossing of the possible σ_+ and σ_- emission energies. Furthermore, at these values of Ω, there are two initial states that are highly occupied with equal probabilities. For this reason there are two peaks in the emission spectra of Fig. 4(c) (for both the σ_+ and σ_- cases) and Fig. 4(g) (for only the σ_+ case) appearing with equal intensities. Above the critical field $\Omega \geq \Omega_c$, the intensity of the σ_- emission decreases as almost all polaritons are in the σ_+ state. The splitting between the σ_+ and σ_- emission peaks increases above Ω_c as the emission energies separate according to Fig. 3.

IV. PHOTOLUMINESCENCE FOR A VARIABLE NUMBER OF POLARITONS

In reality the number of polaritons in the dot can fluctuate due to the spontaneous and stimulated scattering of polaritons into the dot and their radiative escape, so that one should average the PL spectra over N as well. In general, the statistics should depend on the excitation conditions. Here we assume that the polaritons are at quasiequilibrium and one can introduce an effective chemical potential μ so that the distribution is given by

$$p(N,N_+) = Z_G^{-1} e^{[\mu N - E(N,N_+)]/(k_B T)}, \tag{9}$$

where Z_G is the grand partition function.

An important effect due to fluctuations of N is the change in the parity of the polariton number. Polarization steps are expected to be observed for fluctuating N as well, but they should represent a superposition of steps for even and odd N. This way the width of each step becomes $(\alpha_1-\alpha_2)/2$, i.e., a half of the width of the step for the fixed N case. To evidence these "half steps" one should operate with small dots containing a low number of polaritons. The PL spectra for a fluctuating number of polaritons are shown in Figs. 5 and 6. The left hand column of plots show the results for the case $T=20$ K, which can be compared to the left column of Fig. 4. For some values of magnetic fields faint additional peaks can be observed in Fig. 5 corresponding to the mixing of states with different numbers of polaritons. The mixing is more obvious for the case $T=40$ K (see the right hand column of plots in Fig. 5). Note that, in the case of varying numbers of particles, σ_+ and σ_- emission spectra still overlap when Ω is an integer multiple of $(\alpha_1-\alpha_2)/2$ and $\Omega<\Omega_c$. Also, at these values of the field there are pairs of peaks in the emission spectrum appearing with equal intensities. As in the case of a fixed number of polaritons, this is caused by two different initial states having the same probability of occupation. Note that this is in sharp contrast to what we would expect if there were no interactions, in which case only single peaks appear in the spectra (since the emission energy does not depend on the particle number) but with a large energy splitting and intensity difference between σ_+ and σ_- polarizations.

Although there is no well-defined critical field for a fluctuating number of polaritons, the Zeeman splitting is no longer sufficiently suppressed by polariton-polariton interactions at high magnetic fields [see Figs. 5(i) and 5(j), or Fig. 6]. Also, at high magnetic fields, the intensity of the σ_- emission decreases as almost all polaritons are in the σ_+ state. Since the effect of polariton-polariton interactions is to suppress the Zeeman splitting, the intensity of σ_- emission is

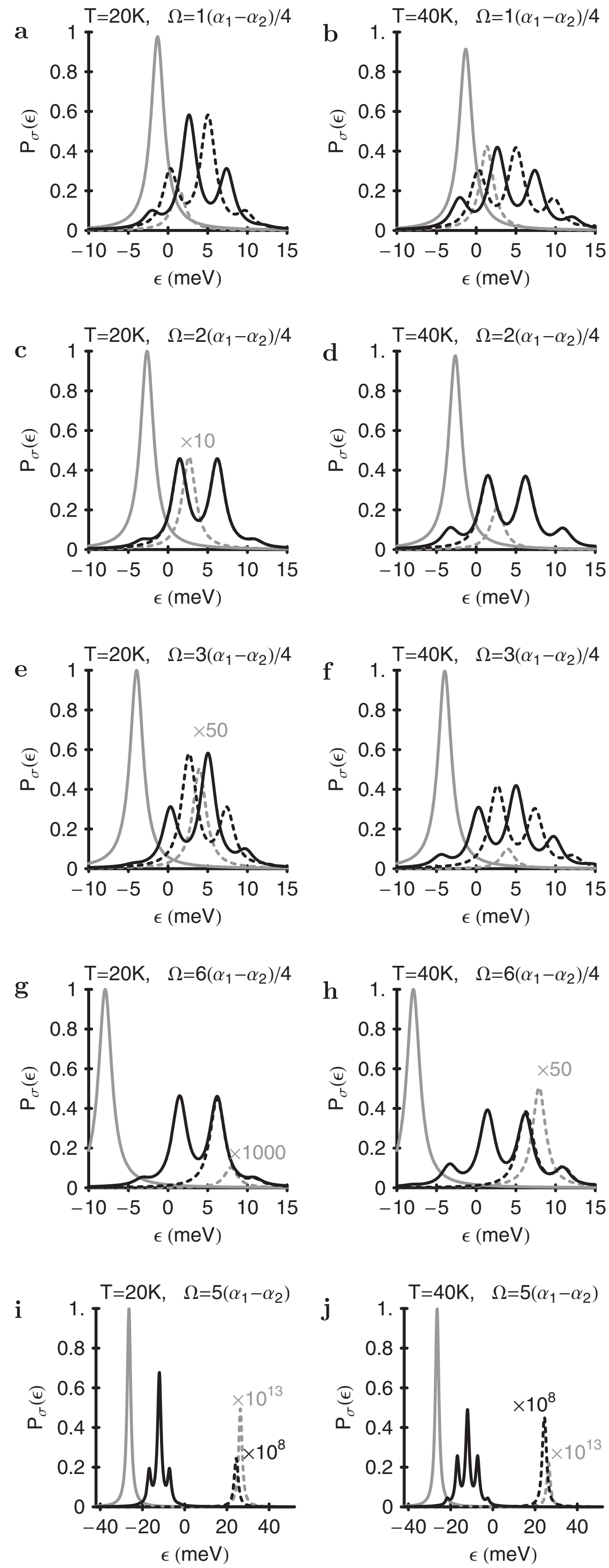

FIG. 5. σ_+ (solid curves) and σ_- (dashed curves) PL spectra for a fluctuating number of polaritons. μ is chosen so that on average there are four polaritons in the dot. Left column: $T=20$ K, right column: $T=40$ K. Grey curves show the spectra in the absence of polariton-polariton interactions. $\Gamma=1$ meV.

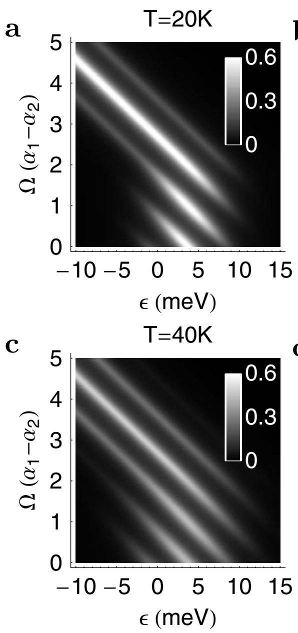

FIG. 6. PL spectra for a fluctuating number of interacting polaritons. μ is chosen so that on average there are four polaritons in the dot. Left column: σ_+ emission; right column: σ_- emission.

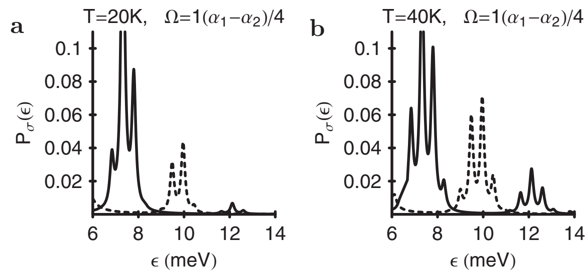

FIG. 7. σ_+ (solid curves) and σ_- (dashed curves) PL spectra for a fluctuating number of polaritons and $\Gamma=0.1$ meV. μ is chosen so that on average there are four polaritons in the dot. (a) $T=20$ K, (b) $T=40$ K.

stronger than that which we would expect if there were no interactions.

It should be noted that there are a relatively small number of peaks seen in the PL spectra compared to what one could expect to observe due to the large number of possible transitions that occur in the case of a fluctuating number of polaritons. This happens because of the small value of the interaction constant α_2 that defines weak attraction between polaritons with opposite spins. In the absence of this attraction, the frequency of a luminescence line in the σ_+ spectrum is defined solely by the initial number N_+ of spin-up polaritons and this frequency is independent of the total polariton number N. Similarly, all transitions with a fixed difference $N-N_+$ contribute to the same line in the σ_- spectrum. The finite value of α_2 results in a fine structure of each peak. This fine structure, however, is not seen in Fig. 5 due to strong lifetime broadening.

To show this fine structure, we plotted the PL spectrum for a much longer polariton lifetime in Fig. 7. One can see the satellite emission around $\epsilon \approx 10$ meV. The faint peaks group to form each emission line. In particular, the faint peaks in the σ_- spectra appear due to the transitions from the states $(N,N_+)=(3,0)$, (4,1), (5,2), (6,3), (7,4), etc. These states have probabilities of occupation equal to 0.001, 0.020, 0.068, 0.056, and 0.012, respectively, for the case $T=40$ K. The emission energies that arise from these states are separated by $|\alpha_2|$.

V. CONCLUSIONS

To summarize, we have shown theoretically that, below the critical magnetic field, governed by the number of polaritons and the difference between polariton-polariton interaction constants in triplet and singlet configurations, the circular polarization degree of a localized polariton condensate increases as a steplike function of the field. For a fixed number of polaritons the width of each step is $\Delta B = (\alpha_1-\alpha_2)/(g\mu_B)$. The values of the field at which steps appear depend on the parity of the total polariton number N, so that the step width is halved when fluctuations in N are present. At finite temperatures, the Zeeman splitting between σ_+ and σ_- emission energies remains close to zero below the critical magnetic field. The polarization steps can be evidenced at finite temperatures in magneto-PL experiments.

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